ENEE2307
Online Quiz Ch3

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The joint probability density function of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 3e^{-(1x+3y)} & 0 \le x, 0 \le y; \\ 0, & \text{otherwise.} \end{cases}$$

Note: $e^1 = 2.718281828$

Determine the $E\{XY\}$. 0.33: *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Here, we are given that,

The joint probability density function of two random variables X and Y is given by
$f_{X,Y}(x,y) = \{ 3e^{-(1x + 3y)}$, $0 \le x$, $0 \le y$
0 , otherwise.

Now, here we find the value of E(XY) as follows.

Since, Given that -

$$f(x,y) = 3e^{-(x+3y)} ; \quad x \geq 0, \; y \geq 0$$

Now we know that

$$E(XY) = \int_X \int_Y xy \; f(x,y) \, dx \, dy$$

$$= \int_0^\infty \int_0^\infty xy \cdot 3 e^{-(x+3y)} \, dx \, dy$$

$$E(XY) = 3 \int_0^\infty x e^{-x} \, dx \int_0^\infty y e^{-3y} \, dy$$

Now, we are using the property of gamma integral -

$$\Rightarrow \quad E(XY) = 3\sqrt{2} \cdot \frac{\sqrt{2}}{3^2} = \frac{3}{9} \quad \left\{ \because \sqrt{2} = 1 \right\}$$

$$E(XY) = \frac{1}{3}$$

$$\Rightarrow \boxed{E(XY) = 0.33333}$$

Thus, this is the required value of $E(XY)$.

Suppose that X and Y have the following joint probability distribution:

Joint Probability Mass Function of X and Y

	Y=1	Y=3	Y=5
X=1	0.1	0.01	0.15
X=2	0.12	0.09	0.53

Determine the mean of X (μ_Y). 1.74 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Determine the mean of Y (μ_Y). 3.92 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Determine the covariance between X and Y ($\mu_{X,Y}$). 6.96 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Solution. Given - Joint PMF of X and Y.

	$Y=1$	$Y=2$	$Y=3$	$P(X=x)$
$X=1$	0.1	0.01	0.15	0.26
$X=2$	0.12	0.09	0.53	0.74
$P(Y=y)$	0.22	0.1	0.68	1.0

$\Rightarrow$ Marginal distribution for X:

$X =$	1	2
$P(X=x)=$	0.26	0.74

$\Rightarrow$ Marginal distribution for Y:

$Y=$	1	2	3
$P(Y=y)$	0.22	0.1	0.68

(i) Mean of X $(\mu_x) = E(X)$

$E(X) = \Sigma x\, P(X=x) = 1 \times P(X=1) + 2 \times P(X=2)$

$= 1 \times 0.26 + 2 \times 0.74$

$\boxed{E(X) = \mu_X = 1.74000}$ is mean of X.

(ii) Mean of Y $(\mu_y) = E(Y)$

$E(Y) = \Sigma y \times P(Y=y) = 1 \times P(Y=1) + 2 \times P(Y=2) + 3 \times P(Y=3)$

$= 1 \times 0.22 + 2 \times 0.1 + 3 \times 0.68$

$= 0.22 + 0.2 + 2.04$

$\boxed{E(Y) = \mu_y = 2.46000}$ is mean of Y.

We know that

$$E(XY) = \sum xy \, P(X=x, Y=y)$$

$$= 1\times1\times P(X=1,Y=1) + 1\times3(P(X=1,Y=3)) + 1\times5\times P(X=1,Y=5)$$
$$+ 2\times1\times P(X=2,Y=1) + 2\times3\times P(X=2,Y=3) + 2\times5(P(X=2,Y=5))$$

$$= 1\times1\times0.1 + 1\times3\times0.01 + 1\times5\times0.15 + 2\times1\times0.12 + 2\times3\times.09$$
$$+ 2\times5\times0.53$$

$$E(XY) = 0.1 + 0.03 + 0.75 + .24 + 0.54 + 5.3$$

$$\boxed{E(XY) = 6.96}$$

Now, $Cov(X,Y) = E(XY) - E(X)\times E(Y)$

$$= 6.96 - 1.74000 \times 2.46000$$

$$= 6.96 - 4.2804$$

$$\boxed{Cov(X,Y) = 2.67960}$$

The joint probability density function of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 9e^{-(3x+3y)} & 0 \le x, 0 \le y; \\ 0, & \text{otherwise.} \end{cases}$$

Note: $e^1 = 2.718281828$

Find the $P(X \le 1.3, Y \le 0.7)$. 0.85! *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Find the $P(X \le 0.4, Y \le 1.2/X \le 3.8)$. 0.67! *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

- $f_{X,Y}(x,y) = 9e^{-(3x+3y)}$; $0 \le x$, $0 \le y$.

$P[X \le 1.3, Y \le 0.7] = \int_0^{1.3}\int_0^{0.7} 9e^{-(3x+3y)}\,dx\,dy$.

This is the joint pdf of two independent exponential random variables

So, $F_{X,Y}(x,y) = F_X(x)F_Y(x)$
$= (1-e^{-\lambda x})(1-e^{-\mu y})$

So, $P(X \le 1.3, Y \le 0.7) = F_{X,Y}(1.3, 0.7)$
$= (1-e^{-1.3\times 3})(1-e^{-0.7\times 3})$
$= \boxed{0.859780}$

- $P[X \le 0.4, Y \le 1.2 \mid X \le 3.8]$

$= \dfrac{P[X \le 0.4 \cap Y \le 1.2 \cap X \le 3.8]}{P[X \le 3.8]}$

$= \dfrac{P[X \le 0.4]\,P[Y \le 1.2]}{P[X \le 3.8]}$

$= \dfrac{(1-e^{-0.4\times 3})(1-e^{-1.2\times 3})}{(1-e^{-(3.8\times 3)})}$

$= \boxed{0.67972}$

The joint probability density function of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} Kxy, & 0 \le x \le 4, 1 \le y \le 4; \\ 0, & \text{otherwise.} \end{cases}$$

Determine the value of the constant K. 0.01 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Solution

Given Joint pdf

$$f_{XY}(x,y) = \begin{cases} k\,xy, & 0 \le x \le 4,\ 1 \le y \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Since $f(x,y)$ is a joint pdf, we have

$$\int_x \int_y f(x,y)\,dy\,dx = 1$$

$$\int_{x=0}^{4} \int_{y=1}^{4} kxy\,dy\,dx = 1$$

$$k \int_{x=0}^{4} x \int_{y=1}^{4} y\,dy\,dx = 1$$

$$k \int_{x=0}^{4} x \left(\frac{y^2}{2}\right)_1^4 dx = 1$$

$$\frac{k}{2} \int_{x=0}^{4} x\,(4^2-1)\,dx = 1$$

$$\frac{k}{2}(16-1) \int_{x=0}^{4} x\,dx = 1$$

$$\frac{15k}{2} \left(\frac{x^2}{2}\right)_0^4 = 1$$

$$\frac{15k}{4} \times 4^2 = 1$$

$$15k \times 4 = 1$$

$$60k = 1 \Rightarrow k = \frac{1}{60}$$

The joint probability density function of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 1e^{-(1x+1y)} & 0 \leq x, 0 \leq y; \\ 0, & \text{otherwise.} \end{cases}$$

Note: $e^1 = 2.718281828$

Find the $F_{X,Y}(0.3, 0.3)$. 0.06: *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

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CHESS

Question

$$f(x,y) = e^{-(x+y)} \qquad x \geq 0,\ y \geq 0$$

$$F(x,y) = \int_0^x \int_0^y f(u,v)\,du.dv.$$

$$= \int_0^x \int_0^y e^{-u} e^{-v}\,du.dv.$$

$$= \int_0^x e^{-u} \int_0^y e^{-v}\,dv\,du.$$

$$= \int_0^x e^{-u} \left[\frac{e^{-v}}{-1}\right]_0^y du.$$

$$= \int_0^x e^{-u}\,(-1)\left[e^{-y} - 1\right] du.$$

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Question → .. cont

$$= (1-e^{-y}) \int_0^x e^{-u} du.$$

$$\Rightarrow (1-e^{-y}) \left(\frac{e^{-u}}{-1} \right)_0^x$$

$$= (1-e^{-y})(1-e^{-x}).$$

$$F_{XY} = (1-e^{-x})(1-e^{-y})$$

$$F_{XY}(0.3, 0.3)$$

$$= (1-e^{-0.3})(1-e^{-0.3}).$$

$$= \left[1-e^{-0.3}\right]^2$$

$$= \left[1-(e)^{-0.3}\right]^2$$

$$= \left[1-(2.718281828)^{-0.3}\right]^2$$

$$\Rightarrow 0.067175194$$

$$F_{X,Y}(0.3, 0.3) = 0.067175194$$

Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

Joint Probability Mass Function of X and Y

	Y=1	Y=3	Y=5
X=1	0.06	0.07	0.04
X=2	0	0.03	0.01
X=3	0.06	0	0.73

Find the $P(X = 3)$. 0.79 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Find the $P(X = 3/Y = 5)$. 0.93 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

Find the $F_{X,Y}(3, 2)$. 0.12 *[The answer should be a number rounded to five decimal places, don't use symbols such as %]*

(1) $P(X=3) = P(X=3 \,\&\, Y=1) + P(X=3 \,\&\, Y=3) + P(X=3 \,\&\, Y=5)$

$P(X=3) = 0.06 + 0 + 0.73$

$P(X=3) = \boxed{0.79}$

(2) $P(X=3 \mid Y=5) = \dfrac{P(X=3 \,\&\, Y=5)}{P(Y=5)}$

$= \dfrac{0.73}{0.73+0.01+0.04}$

$P(X=3 \mid Y=5) = \dfrac{0.73}{0.78} = \boxed{0.93590}$

(3) $F_{XY}(3,2) = P(X \leq 3)\; P(Y \leq 2)$

$= (1)\,(0.12)$

$= \boxed{0.12}$

Let X denote the diameter of an armored electric cable and Y denote the diameter of the ceramic mold that makes t
cable. Both X and Y are scaled so that they range between 0 and 2. Suppose that X and Y have the joint density

$$f(x, y) = \begin{cases} Ky & 0 < x < y < 2; \\ 0 & \text{otherwise.} \end{cases}$$

1. Determine the value of the constant K.
2. Are X and Y statistically independent?
3. Determine the $P(X + Y > 0.5)$.

STAT

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Ans: $\int_0^y \int_0^x f_{xy}(x,y)\, dx\, dy = 1$

$\int_0^2 \int_0^y K y\, dx\, dy = 1$

$\int_0^2 \int_0^y y\, dx\, dy = \frac{1}{K}$

$\int_0^2 y\, x \Big]_0^y\, dy = \frac{1}{K}$

$\int_0^2 y^2\, dy = \frac{1}{K}$

$\frac{y^3}{3} \Big]_0^2 = \frac{1}{K} \longrightarrow$

$$\rightarrow \frac{8}{3} = \frac{1}{K}$$

$$\rightarrow K = \frac{3}{8}$$

② $f(x) = \int_0^x f(x,y)\, dy$

$$= \int_0^x \frac{3}{8} y \, dy$$

$$= \frac{3}{8} \frac{y^2}{2} \Big]_0^x$$

$$= \frac{3}{16} x^2$$

$$f(y) = \int_0^y f(x,y)\, dx$$

$$= \int_0^y \frac{3}{8} y \, dx$$

$\longrightarrow$

$$\rightarrow \frac{3}{8} y\, x \Big]_0^y$$

$$= \frac{3}{8} y^2$$

$$f(x)\, f(y) = \frac{3}{16} x^2 \cdot \frac{3}{8} y^2$$

$$= \frac{9}{128} x^2 \cdot y^2$$

$$f(x)\, f(y) \neq f(x, y)$$

$\therefore$ x and y are not statically independent.

③ $P(X+Y > 0.5)$

$$= \int_0^{0.25} \int_X^{0.5-X} \frac{3}{8} y \, dy \, dx$$

$$= \int_0^{0.25} \frac{3}{16} y^2 \Big]_x^{0.5-x} dx$$

$$= \frac{3}{16} \int_0^{0.25} (0.5-x)^2 - x^2 \, dx$$

$$= \frac{3}{16} \int_0^{0.25} (0.25-x) \, dx$$

$$= \frac{3}{16} \left(0.25x - \frac{x^2}{2} \Big]_0^{0.25} \right)$$

$$= \frac{3}{16} \left((0.25)^2 - \frac{(0.25)^2}{2} \right)$$

$$= \frac{3}{16} \frac{(0.25)^2}{2} = 5.859 * 10^{-3}$$